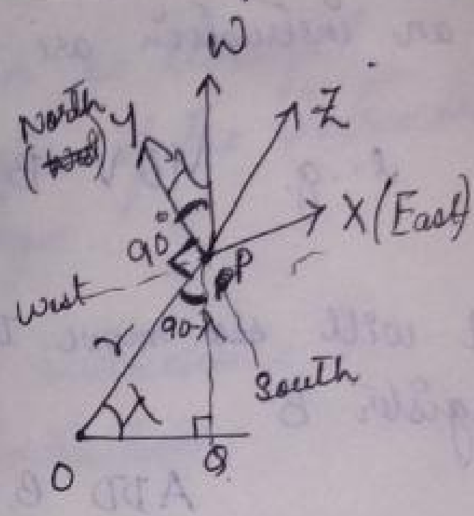
Effect of Coriollis Force on a particle moving on the surface of earth



$\angle OPQ = 90 - \lambda$

→ Y is tangent.

→ r is position vector

$\therefore r \perp PY$

$\therefore \angle OPY = 90°$

$\therefore \angle YPW = \lambda$.

$[\because 90 - \lambda + 90 + A = 180°]$

$\Rightarrow A = \lambda$.

Let us imagine a particle moving on the surface of the earth with a velocity v.

(Surface of the earth means moving in XY plane)

$$\therefore \vec{v} = v_x \hat{i} + v_y \hat{j}.$$

Angular velocity of earth of point P

$$\vec{\omega} = \omega \cos\lambda \hat{j} + \omega \sin\lambda \hat{k}.$$

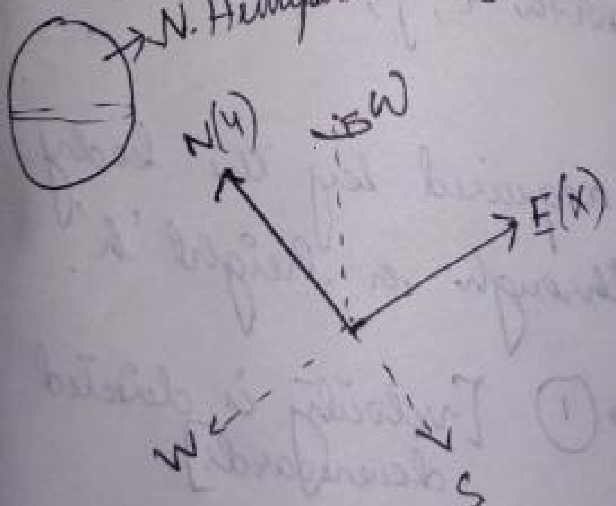
$\therefore$ Coriolis Force, $\vec{F}_c = -2m(\vec{\omega} \times \vec{v})$

$$= -2m(\omega\cos\lambda\,\hat{j} + \omega\sin\lambda\,\hat{k}) \times (v_x\hat{i} + v_y\hat{j})$$

$$= \underbrace{2m\omega\cos\lambda\, v_x\hat{k}}_{\text{Vertical force}} + \underbrace{2m\omega\sin\lambda\,(v_y\hat{i} - v_x\hat{j})}_{\text{Horizontal force}}$$

Effect of horizontal acc^n on a body moving in Northern Hemisphere.

① Body is moving along X-axis (East)



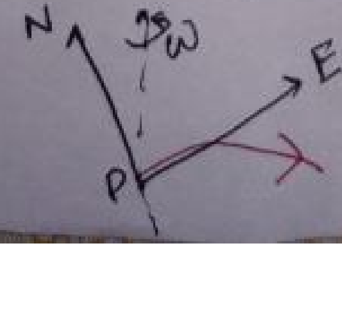
i.e. $v_y = 0$.

$\therefore F_c = -2m\omega\sin\lambda\, v_x\hat{j}$

The force is along $-\hat{j}$

i.e. along South direction

$\therefore$ A particle moving in X-direction in the northern hemisphere will deviate along South direction.



(2) Body moving along N-direction.

$$v_x = 0$$

$$\vec{F} = 2m\omega \sin\lambda \, v_y \, \hat{i}$$

i.e. deflection along '+x' drxn i.e. it deviates towards East.

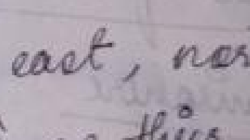
Effect of Coriolis Force on a particle falling freely under gravity :-

Let us take the axes x, y, z along east, north and vertically upwards respectively with $\hat{i}, \hat{j}, \hat{k}$ as their unit vectors.

Let 'v' be the velocity acquired by the body in time 't' taken by it to fall through a height 'h'.

i.e. $\vec{v}' = -v\hat{k} \rightarrow (1)$ [velocity is directed downwards]

Now $\vec{\omega} = \omega\cos\lambda \, \hat{j} + \omega \sin\lambda \, \hat{k}$.



where λ is the latitude of the place.

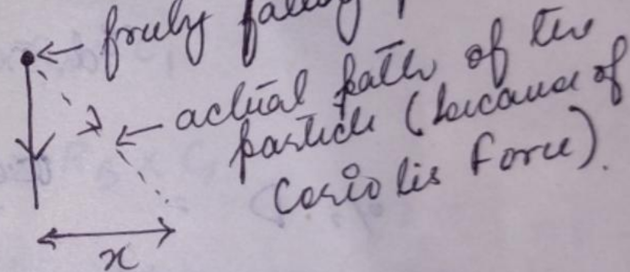
Coriolis acceleration $\vec{a}_c = -2\,\vec{\omega}\times\vec{v}'$

$$= -2(\omega\cos\lambda\,\hat{j} + \omega\sin\lambda\,\hat{k}) \times (-v'\hat{k})$$

$$= 2\omega v\cos\lambda\,\hat{i}.$$

$\therefore$ Coriolis Force, $F_c = 2m\omega v\cos\lambda\,\hat{i}.$

Thus the freely falling particle suffers a deflection from its truly vertical path.



Now

$$F_c = m\frac{d^2x'}{dt^2}$$

or $2\cancel{m}\omega v'\cos\lambda\,\hat{i} = \cancel{m}\frac{d^2x'}{dt^2}$ | $v = a\,t$, $a = \frac{v}{t}$

$\Rightarrow \frac{d^2x'}{dt^2} = 2\omega v'\cos\lambda\,\hat{i}$

Again, $v' = gt$, where g is the accel^n due to gravity

$\therefore \frac{d^2x'}{dt^2} = 2\omega gt\cos\lambda$

or $v_x' = \frac{dx}{dt} = \int 2\omega gt\cos\lambda\,dt$

$= 2\omega g\cos\lambda\frac{t^2}{2} + c$

Now at $t=0$, $v_x = 0$ which gives $c = 0$

$\therefore v_x' = \omega g\cos\lambda\,t^2$

Integrating once again, we get.

Displacement along x-axis

i.e. $x = \int \omega g\cos\lambda\,t^2\,dt$

$= \omega g\cos\lambda\frac{t^3}{3} + c$

Again at $t=0$, there is no displacement i.e. $x=0$

$\therefore c' = 0$

$$\therefore x = \frac{1}{3}\omega g \cos\lambda\, t^3$$

Now, if t is the time taken by the body to fall through a height h then

$$h = \frac{1}{2}gt^2 \quad \left[\because \text{initial velocity } u=0\right]$$

$$\Rightarrow t = \sqrt{\frac{2h}{g}}$$

$$\therefore x = \frac{1}{3}\omega g\cos\lambda\left(\frac{2h}{g}\right)^{3/2}$$

$$\boxed{x = \left(\frac{8}{9g}\right)^{1/2} h^{3/2}\,\omega\cos\lambda}$$

↳ horizontal displacement of the body at latitude λ due to coriolis force.

For $\lambda = 0$ i.e. at the equator $\cos\lambda = 1$

& $x = \left(\frac{8}{9g}\right)^{1/2} h^{3/2}\,\omega$ i.e. maximum.